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Decision Rule

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In the context of statistical hypothesis testing, decision rule refers to the rule that specifies how to choose between two (or more) competing hypotheses about the observed data. A decision rule specifies the statistical parameter of interest, the test statistic to calculate, and how to use the test statistic to choose among the various hypotheses about the data. More broadly, in the context of statistical decision theory, a decision rule can be thought of as a procedure for making rational choices given uncertain information.

The choice of a decision rule depends, among other things, on the nature of the data, what one needs to decide about the data, and at what level of significance. For instance, decision rules used for normally distributed (or Gaussian) data are generally not appropriate for non-Gaussian data. Similarly, decision rules used for determining the 95% confidence interval of the sample mean will be different from the rules appropriate for binary decisions, such as determining whether the sample mean is greater than a prespecified mean value at a given significance level. As a practical matter, even for a given decision about a given data set, there is no unique, universally acceptable decision rule but rather many possible principled rules.

There are two main statistical approaches to picking the most appropriate decision rule for a given decision. The classical, or frequentist, approach is the one encountered in most textbooks on statistics and the one used by most researchers in their data analyses. This approach is generally quite adequate for most types of data analysis. The Bayesian approach is still widely considered esoteric, but one that an advanced researcher should become familiar with, as this approach is becoming increasingly common in advanced data analysis and complex decision making.

Decision Rules in Classical Hypothesis Testing

Suppose one needs to decide whether a new brand of bovine growth hormone increases the body weight of cattle beyond the known average value of μ kilograms. The observed data consist of body weight measurements from a sample of cattle treated with the hormone. The default explanation for the data, or the null hypothesis, is that there is no effect: the mean weight of the treated sample is no greater than the nominal mean μ. The alternative hypothesis is that the mean weight of the treated sample is greater than μ.

The decision rule specifies how to decide which of the two hypotheses to accept, given the data. In the present case, one may calculate the t statistic, determine the critical value of t at the desired level of significance (such as .05), and accept the alternative hypothesis if the t value based on the data exceeds the critical value and reject it otherwise. If the sample is sufficiently large and Gaussian, one might use a similar decision rule with a different test statistic, the z score. Alternatively, one may choose between the hypotheses based on the p value rather than the critical value.

Such case-specific variations notwithstanding, what all frequentist decision rules have in common is that they arrive at a decision ultimately by comparing some statistic of the observed data against a theoretical standard, such as the sampling distribution of the statistic, and determine how likely the observed data are under the various competing hypotheses.

Conceptual quibbles about this view of probability aside, this approach is entirely adequate for a vast majority of practical purposes in research. But for more complex decisions in which a variety of factors and their attendant uncertainties have to be considered, frequentist decision rules are often too limiting.

Bayesian Decision Rules

Suppose, in the aforementioned example, that the effectiveness of the hormone for various breeds of cattle in the sample, and the relative frequencies of the breeds, is known. How should one use this prior distribution of hormone effectiveness to choose between the two hypotheses? Frequentist decision rules are not well suited to handle such decisions; Bayesian decision rules are.

Essentially, Bayesian decision rules use Bayes's law of conditional probability to compute a posterior distribution based on the observed data and the appropriate prior distribution. In the case of the above example, this amounts to revising one's belief about the body weight of the treated cattle based on the observed data and the prior distribution. The null hypothesis is rejected if the posterior probability is less than the user-defined significance level.

One of the more obvious advantages of Bayesian decision making, in addition to the many subtler ones, is that Bayesian decision rules can be readily elaborated to allow any number of additional considerations underlying a complex decision. For instance, if the larger decision at hand in the above example is whether to market the hormone, one must consider additional factors, such as the projected profits, possible lawsuits, and costs of manufacturing and distribution. Complex decisions of this sort are becoming increasingly common in behavioral, economic, and social research. Bayesian decision rules offer a statistically optimal method for making such decisions.

It should be noted that when only the sample data are considered and all other factors, including prior distributions, are left out, Bayesian decision rules can lead to decisions equivalent to and even identical to the corresponding frequentist rules. This superficial similarity between the two approaches notwithstanding, Bayesian decision rules are not simply a more elaborate version of frequentist rules. The differences between the two approaches are profound and reflect longstanding debates about the nature of probability. For the researcher, on the other hand, the choice between the two approaches should be less a matter of adherence to any given orthodoxy and more about the nature of the decision at hand.

See also Criterion Problem; Critical Difference; Error Rates; Expected Value; Inference: Inductive and Deductive; Mean Comparisons; Parametric Statistics

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